

Numerical results from large N reduced QCD₂

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Some results in QCD₂ at large N are presented using the reduced model on the lattice. Overlap fermions are used to compute meson propagators.

1. Introduction

This follows the talk given by H. Neuberger [1] and summarizes the numerical results obtained in QCD₂ at large N using the quenched reduced model and overlap fermions [2].

The reduced model has a single site, and there are d SU(N) matrices $U_\mu, \mu = 1, \dots, d$. The gauge action is given by

$$S_g = -\beta \sum_{\mu > \nu} \text{Tr} C_{\mu\nu} C_{\mu\nu}^\dagger; \quad C_{\mu\nu} = [U_\mu, U_\nu], \quad (1)$$

where $\beta = \frac{1}{2g^2}$ and g^2 is the continuum gauge coupling. We will keep $b^2 = \frac{\beta}{N}$ fixed as N goes to infinity, and this amounts to setting $g^2 N = \frac{1}{2b^2}$.

The reduced model on the lattice has a $[U(1)]^d$ symmetry. The Eguchi-Kawai argument for reduction holds only if the $[U(1)]^d$ symmetry remains unbroken [3]. This is not the case as $b^2 \rightarrow \infty$ [4]. This problem is resolved by defining the quenched reduced model [4] where the eigenvalues of U_μ ,

$$U_\mu = V_\mu D_\mu V_\mu^\dagger; \quad D_\mu = \text{diag}(e^{i\theta_\mu^1}, e^{i\theta_\mu^2}, \dots, e^{i\theta_\mu^N}), \quad (2)$$

are fixed, and V_μ s are the only dynamical degrees of freedom. The θ_μ^i are randomly distributed in the interval $[-\pi, \pi]$, and a quenched average over these variables is taken. However in QCD₂, it is

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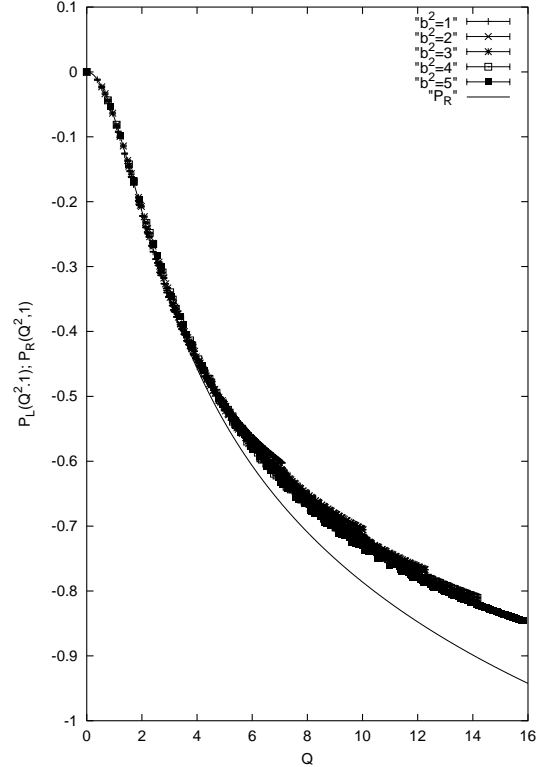


Figure 1. Plot of $P_L(Q^2, 1)$ at $N_c = 41$ for $b^2 = 1, 2, 3, 4, 5$.

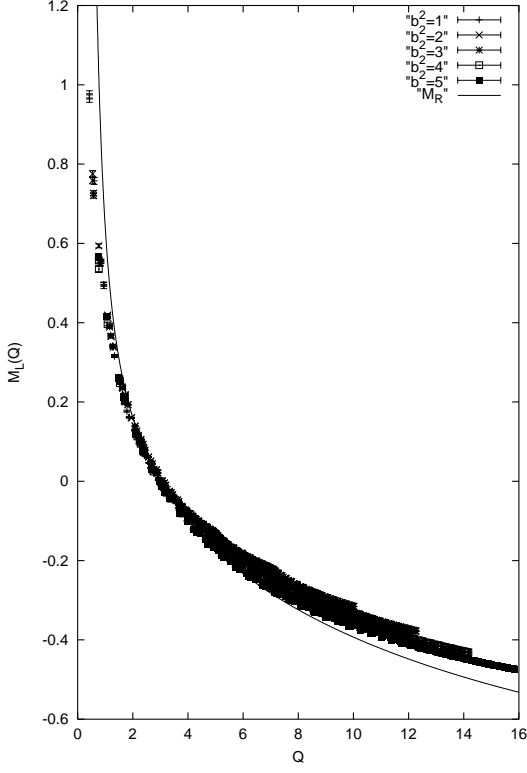


Figure 2. Plot of the meson propagator for massless quarks as a function of b^2 . The $b^2 = 1$ data is at $N_c = 37$, the $b^2 = 2, 4$ data are at $N_c = 41$ and the $b^2 = 3, 5$ data are at $N_c = 47$.

sufficient to pick θ_μ^i to be the N roots of unity, and there is no need to perform a quenched average [4]. Minima of the action occur when all the U_μ s are simultaneously diagonal. Certain V_μ s relate one minimum with another minimum. Numerical simulations should sample all minima properly.

Fermionic loops are suppressed by $1/N$ in the large N limit, and only the gauge degrees of freedom determine the dynamics. One can compute the fermionic propagator in momentum space by force feeding momenta [5]. This amounts to using $e^{ip_\mu} U_\mu$ for the fermion parallel transporter in the μ direction which carries a momentum p_μ . We

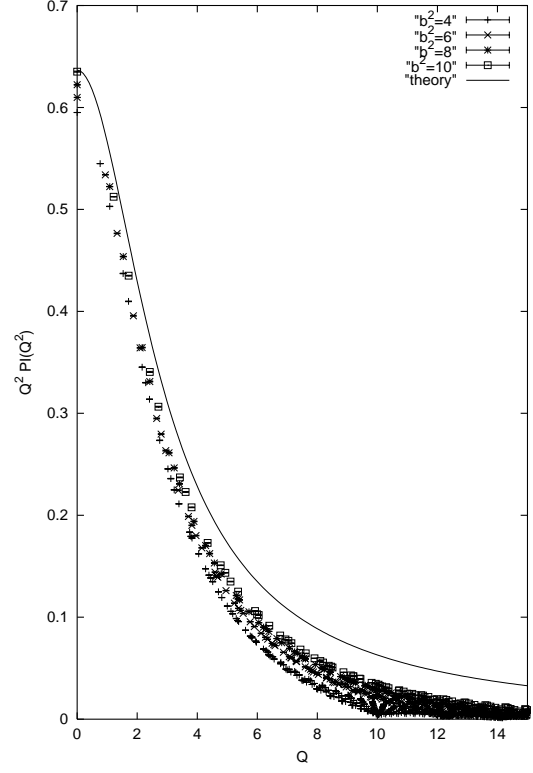


Figure 3. Plot of the current-current correlator at $\gamma = 1$ as a function of b^2 at $N = 41$.

will use the propagator for massive overlap Dirac operator given by [6]

$$g(p) = \frac{1}{2m} \frac{1 - \gamma_{d+1} \epsilon(H_w)}{(1 + \mu) + (1 - \mu) \gamma_{d+1} \epsilon(H_w)}. \quad (3)$$

The quark mass is $m_q = 2m\mu$ where m is the magnitude of the negative Wilson mass that appears inside H_w , the hermitian Wilson-Dirac operator.

2. Correlation functions

Meson correlation functions are known analytically in QCD₂ [7], and we can check the results obtained from the quenched reduced model on the lattice. Correlators can be expressed as an infinite sum over stable mesons, which reproduce free field behavior at large momenta. Therefore

we have confinement and asymptotic freedom in QCD₂. For $m_q = 0$, there is a massless meson in the spectrum, which is interpreted as a Goldstone boson. Chiral symmetry breaking occurs if we take $m_q \rightarrow 0$ after we take $N \rightarrow \infty$.

Gauge fields are generated using (1) and the quenching condition in (2). The eigenvalues of U_1 and U_2 are set to the N roots of unity, and gauge field configurations are generated at a fixed b^2 for different values of N . Fermionic momenta also are fixed at N roots of unity. The “bare” scalar and pseudoscalar propagators are computed using

$$\begin{aligned}
S_0(P) &= \frac{1}{4N^2} \sum_{p_1, p_2} \\
&\quad Tr[g(p_1 + P_1, p_2 + P_2)g(p_1, p_2) \\
&\quad + g(p_1 - P_1, p_2 + P_2)g(p_1, p_2) \\
&\quad + g(p_1 + P_2, p_2 + P_1)g(p_1, p_2) \\
&\quad + g(p_1 + P_2, p_2 - P_1)g(p_1, p_2)] \\
P_0(P) &= \frac{1}{4N^2} \sum_{p_1, p_2} \\
&\quad Tr[g(p_1 + P_1, p_2 + P_2)g^\dagger(p_1, p_2) \\
&\quad + g(p_1 - P_1, p_2 + P_2)g^\dagger(p_1, p_2) \\
&\quad + g(p_1 + P_2, p_2 + P_1)g^\dagger(p_1, p_2) \\
&\quad + g(p_1 + P_2, p_2 - P_1)g^\dagger(p_1, p_2)]
\end{aligned}$$

This results in a correlation function for mesons that is a function of one variable, namely, $P^2 = 4(\sin^2 \frac{P_1}{2} + \sin^2 \frac{P_2}{2})$. The scale is set by $e^2 = \frac{1}{2\pi b^2}$, and we will use $\gamma = \frac{m_q^2}{e^2}$ as a dimensionless measure of the quark mass. We will also use a dimensionless momentum $Q^2 = \frac{P^2}{e^2}$. The bare correlators have to be regularized. This is done by a subtraction at zero momentum for massive quarks and a subtraction at non-zero momentum for massless quarks. We will refer to the renormalized quantities on the lattice by $S_L(Q^2, \gamma)$ and $P_L(Q^2, \gamma)$. In the dimensionless notation, we have to compare these results on the lattice with the regularized results in the continuum

$$P_R(Q^2, \gamma) = \sum_{n \geq 0 \text{ even}}^{\infty} \left[\frac{r_n^2}{Q^2 + \mu_n^2} - \frac{r_n^2}{\mu_n^2} \right] \quad (4)$$

$$S_R(Q^2, \gamma) = \sum_{n \geq 1 \text{ odd}}^{\infty} \left[\frac{r_n^2}{Q^2 + \mu_n^2} - \frac{r_n^2}{\mu_n^2} \right]. \quad (5)$$

The dimensionless meson masses μ_n are obtained by solving the 't Hooft Hamiltonian [2], and $\mu_n^2 \sim \pi^2 n$ in the asymptotic limit. The residues, r_n^2 , are given by

$$r_n^2 = \frac{\gamma}{\pi} \left[\int_0^1 dx \frac{\phi_n(x)}{x} \right]^2 \quad (6)$$

where $\phi_n(x)$ are the eigenvectors of the 't Hooft Hamiltonian.

Figure 1 shows the results for $P_L(Q^2, 1)$ for various values of b^2 and the comparison to the continuum result. One can also compute the correlation function for massless quarks at finite N . In this case, one will not see the effect of chiral symmetry breaking. The propagators for the scalar and the pseudoscalar are the same and should match with the average of the two in the continuum. Figure 2 shows the results for massless quarks along with the continuum result. One can also compute current-current correlators and obtain $\Pi(Q^2)$. We present the results for current-current correlators at $\gamma = 1$ along with the continuum result in figure 3.

3. Conclusions

The numerical results obtained in QCD₂ using the quenched reduced model and overlap fermions are promising. The next step is to perform the same computation for QCD₄, and this project is under way.

REFERENCES

1. J. Kiskis, R. Narayanan and H. Neuberger, Talk presented by H. Neuberger in these proceedings.
2. J. Kiskis, R. Narayanan and H. Neuberger, Phys. Rev. D66, 025019 (2002).
3. T. Eguchi and H. Kawai, Phys. Rev. Lett. 48, 1063 (1982).
4. G. Bhanot, U.M. Heller and H. Neuberger, Phys. Lett. B112, 47 (1982).
5. H. Levine and H. Neuberger, Phys. Lett. B119, 183 (1982).
6. R.G. Edwards, U.M. Heller and R. Narayanan, Phys. Rev. D59, 094510 (1999).
7. G. 't Hooft, Nucl. Phys. B75, 461 (1974).